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Original Article

Mathematical Model Transmission Dynamics of Onychomycosis with Cymbopogon Citratus Application as Control on Patients in Selected Hospitals in Benue State, Nigeria

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Abstract

Onychomycosis, a fungal nail infection, presents significant public health challenges, including in Benue State, Nigeria. Traditional plants like Cymbopogon citratus (lemon grass) have demonstrated antifungal properties. This study develops a mathematical model to analyze onychomycosis transmission dynamics and evaluate C. citratus extracts as a potential treatment. The model uses ordinary differential equations, categorizing individuals as susceptible (S), infected (I), and recovered (R) over time. Simulations show that reducing contact with infected individuals lowers transmission rates. Consistent treatment with C. citratus increases recovery, thereby decreasing susceptibility. Numerical simulations reveal that applying C. citratus extracts using a sterilized nail polish brush over two years resulted in reduced infection. The treatment regimen involved bi-weekly application for up to two years, with periodic monitoring of nail improvement. The study highlights that isolating infected individuals, combined with treatment and environmental precautions (such as avoiding shared nail equipment and walking barefoot in contaminated areas), significantly reduces transmission. These findings suggest that early detection, isolation, and consistent treatment with C. citratus are key to managing and potentially eradicating onychomycosis in the population.

Keywords: Cymbopogon Citratus, Mathematical Model, Onychomycosis, Benue State.

1. Introduction

Onychomycosis, a fungal infection of the nails, is a common condition worldwide. The infection is caused by various fungi, including dermatophytes, yeasts, or non-dermatophyte molds (de Berker, 2009). This condition can significantly impact patients' emotional, social, and occupational functioning. Patients with onychomycosis may feel embarrassed in social and workplace situations, unwilling to expose their hands or feet due to the perception of being blighted or unclean.

Several factors may predispose individuals to onychomycosis, including diabetes mellitus, age, nail trauma, poor peripheral circulation, immunosuppression, tightly fitting shoes, long nails, going barefoot in public places, sharing shoes, and exposing hands or feet to damp environments. Various treatments are available for onychomycosis, including oral antifungal drugs, medicated nail polish, creams, nail removal, and the use of medicinal plants (Westerberg & Voyack, 2013).

The use of medicinal plants in the treatment of onychomycosis has gained attention in recent years due to concerns about the side effects of conventional antifungal drugs (Girois et

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al., 2006). One such plant is *Cymbopogon citratus*, locally known as lemongrass, Toho Gwaza in Tiv dialect of Benue State. Lemongrass has been used traditionally for various medicinal purposes, including its antifungal properties. Tyona & Musa (2024) reported the phytocompound screening, cytotoxicity screening and antifungal activity of *Cymbopogon citratus* extracts against onychomycosis pathogens, including Trichophyton *rubrum, Epidemorphyton floccosum, Candida* spp, *Aspergillus* spp, and *Trichophyton interdigitale*. Onychomycosis infection results from a combination of processes that range widely in scale: from the biological molecules involved in the intracellular metabolism of pathogens to the interactions between individual people who may host and spread onychomycosis across the population. Despite the wide range of scales, in each of these processes the behavior of the system results from the interactions between many elements. Understanding how each element works in isolation is important. However, possessing such knowledge may not be adequate if the behavior of the entire system arises as an emergent property from the interactions between its components (López-Benítez & Casadevall, 2011). Mathematical modeling provides a way to describe and quantify interactions among elements and dissect their relative contribution to infection dynamics (João et al., 2022).

The objective of this study was to develop and authenticate a mathematical model for the transmission and management of onychomycosis, employing a system of first-order ordinary differential equations comprising three distinct compartments.

2. Methods

The study focuses on a sample population of 384 adults from Benue State diagnosed with onychomycosis. Participants are treated with *Cymbopogon citratus* extract, and the progression of their condition is monitored over a period of 0-2 years. The mathematical model was formulated using compartmental analysis modeling approach of susceptible (S), infected (I) and recovered (R); (SIR) and represented in the form of a system of first order ordinary differential equations (ODEs) (Schlosser, 2020).

2.1. Assumptions of the model

The model was formulated using the following assumptions as described by João et al. (2022):

a) The population of the study, N is equivalent to the sample size as determined in the study. This population is sub-divided into Susceptible (S), Infected (I) and Recovered (R) individuals in a manner that N = S + I + R.
b) The epidemiological classes in assumption i above depends on the time (t) take N for Onychomycosis to spread in the environment under study.
c) Onychomycosis can be transmitted among family members via shared surfaces, shared equipment and contaminated environments.
d) The exposure of individuals to the behavioral risk factors enshrined in the study can lead to the spread of Onychomycosis.
e) The susceptible population could be subjected to prevention and control to reduce the rate of infectivity of the disease while treatment control is imposed on the infected population to improve on the number of recovered individuals.
f) There is every tendency for re-infection; hence the recovered population has to be enlightened about their state of infectivity.

2.2. Variables and parameters of the model

The variables and parameters of the model are as shown in Table 1 and Table 2 respectively (João et al., 2022).

Table 1. Model flow showing the variables of the model

S/No.	Variable	Meaning
1	t	Time taken for the infection to spread
2	$S(t)$	Susceptible to
3	$I(t)$	onychomycosis Infected
4	$R(t)$	Recovered individuals
5	$N(t)$	Total population
6	$u_1(t)$	Prevention control
7	$u_2(t)$	Treatment control

Source: João et al. (2022)

Table 2. Model flow showing the parameters of the model

S/No.	Variable	Meaning
1	α	Recruitment rate of onychomycosis
2	b	Infectivity rate
3	γ	Rate of re-infection
4	δ	Recruitment rate of infected individuals
5	Σ	Susceptibility rate of recovery
6	T	Recovery rate

Source: João et al. (2022)

2.3. Model flow diagrams

There are two model flow diagrams constructed as seen in Figures 1, 2 and 3 representing the transmission dynamics.

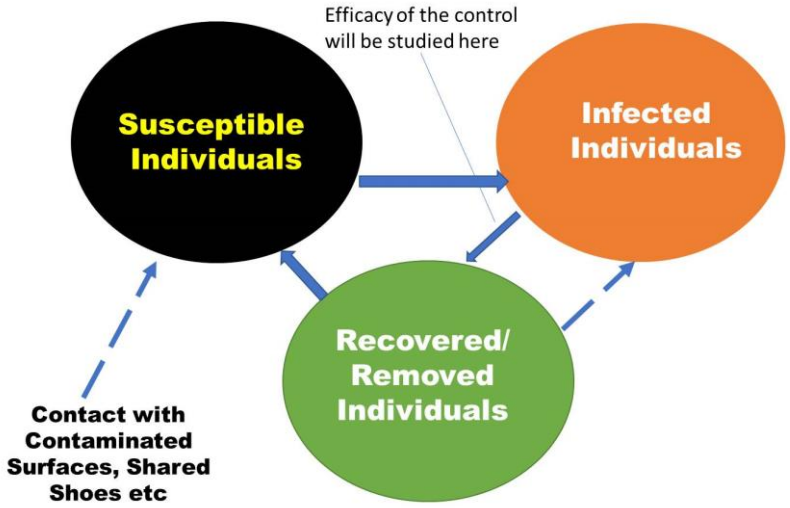

Figure 1. Schematic Flow of the Model Flow Process

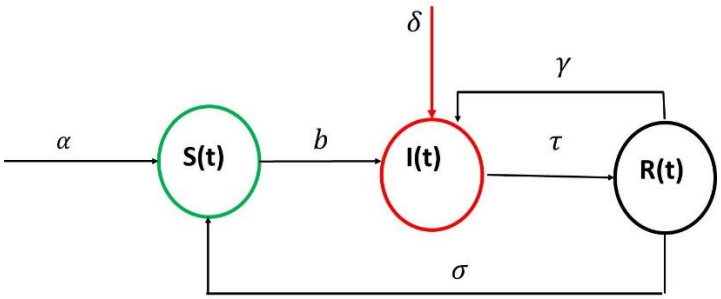

Figure 2. Model Flow Diagram showing the transmission dynamics of onychomycosis without control measures

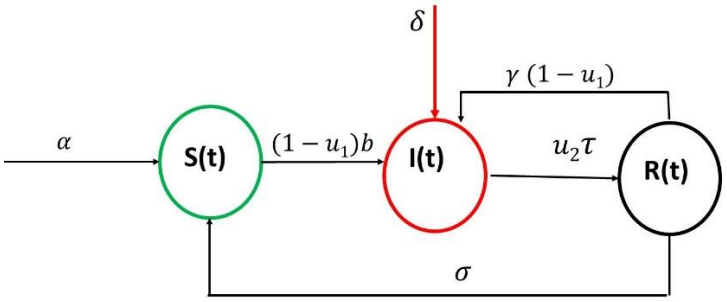

Figure 3. Model flow diagram showing the transmission dynamics of onychomycosis with the presence of control measures (prevention and treatment controls)

2.4. The governing mathematical models

The mathematical models governing the transmission dynamics of onychomycosis without control as presented in Figure 1a) is given by equation (1) following methods by McQuade et al. (2017).

$$\left.\begin{aligned} \frac{dS(t)}{dt} &= \alpha N(t) + \sigma R(t) - bI(t)S(t) \\ \frac{dI(t)}{dt} &= bI(t)S(t) + \gamma R(t) + \delta I - \tau I(t) \\ \frac{dR(t)}{dt} &= \tau I(t) + (\sigma + \gamma)R(t) \end{aligned}\right\} \quad (1)$$

While a mathematical model governing the transmission dynamics of onychomycosis and the presence of control measures (prevention and treatment controls) as enshrined in the model flow diagram in Figure 1b) is governed by equations (2);

$$\left.\begin{aligned} \frac{dS(t)}{dt} &= \alpha N(t) + \sigma R(t) - b(1-u_1)I(t)S(t) \\ \frac{dI(t)}{dt} &= (1-u_1)bI(t)S(t) + (1-u_1)\gamma R(t) + \delta I - u_2\tau I(t) \\ \frac{dR(t)}{dt} &= u_2\tau I(t) - (\sigma + (1-u_1)\gamma)R(t) \end{aligned}\right\} \quad (2)$$

Where

$N(t) = S(t) + I(t) + R(t)$

2.5. Method of data analysis

The solution of the model was obtained using the ordinary differential equation (ODE45) module in MATLAB built based on the Runge-Kutta 4th and 5th – order methods.

3. Results and Discussion

3.1. Research Results

3.1.1. Model Results

The model was developed using the laboratory results of onychomycosis infection as presented in table 3.

Table 3. Direct Microscopy (KOH) and Cultures

Direct Microscopy	Culture Positive (%)	Culture Negative (%)	Total (%)
Positive	195 (92.86%)	15 (7.14%)	210 (100%)
Negative	31 (17.82%)	143 (82.18%)	174 (100%)
Total	226 (58.85%)	158 (41.15%)	384 (100%)

The results present the initial conditions and parameter values used for the simulation of the model as shown in table 4.

Table 4. Model variables and their values

S/No.	Variable	Value	Source
1	t	t ∈ [0,2]	Experiment
2	$S(t)$	384	Experiment
3	$I(t)$	226	Experiment
4	$R(t)$	158	Estimated
5	$N(t)$	S+I+R	Estimated
6	$u_1(t)$	$u_1(t) \in [0,1]$	Standard
7	$u_2(t)$	$u_2(t) \in [0,1]$	Standard

Table 5. Model parameters and their values

S/No.	Variable	Value	Source
1	α	0.550000	Estimated
2	b	0.588542	Estimated
3	γ	0.150000	Estimated
4	δ	0.450000	Estimated
5	Σ	0.250000	Estimated
6	T	0.411458	Estimated

3.1.1. Results of the Numerical Simulations

The results of the study are plotted and represented in Figure 4.

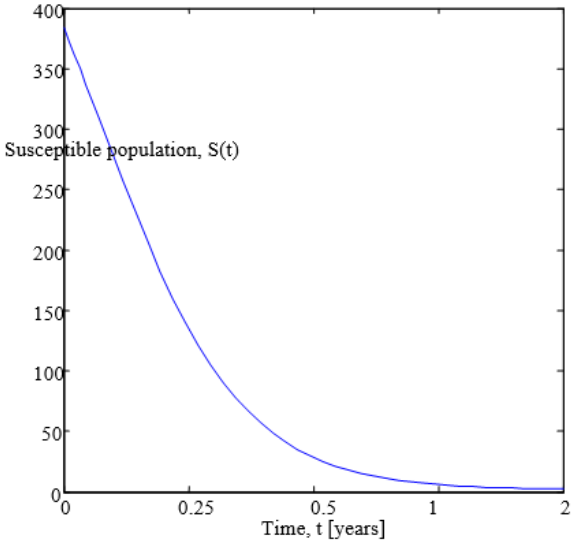

Figure 4. Population of Susceptible Individuals, S (t) without control in 1 year, i.e. $t \in [0, 2]$ year

Figure 4 as onychomycosis increases, the population infection rate increases. This implies onychomycosis rate, has the direct impact on the infected population. It is therefore important to consider all necessary precaution and treatment needed to control the increase in infection rate of the disease.

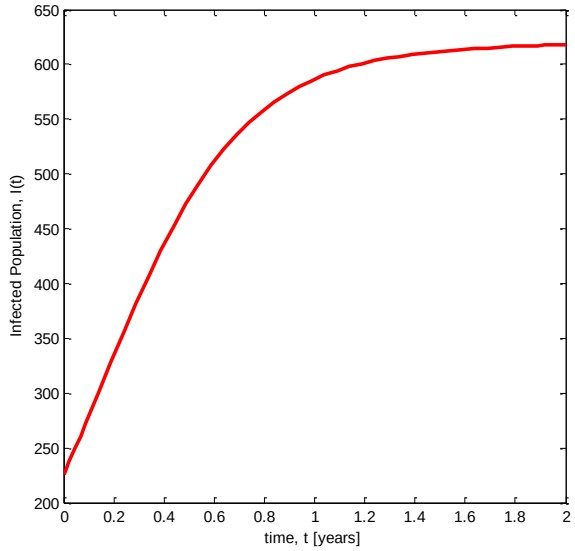

Figure 5. Population of Infected Individuals, I(t) without control within two years, i.e. $t \in [0, 2.00]$ years

Figure 5 show that there is a massive increase rate of onychomycosis infection in the population without treatment using *C. citratus*. More so if there is high interaction with the contaminated environment, there was high increase in the susceptibility and infected individuals over several months to years.

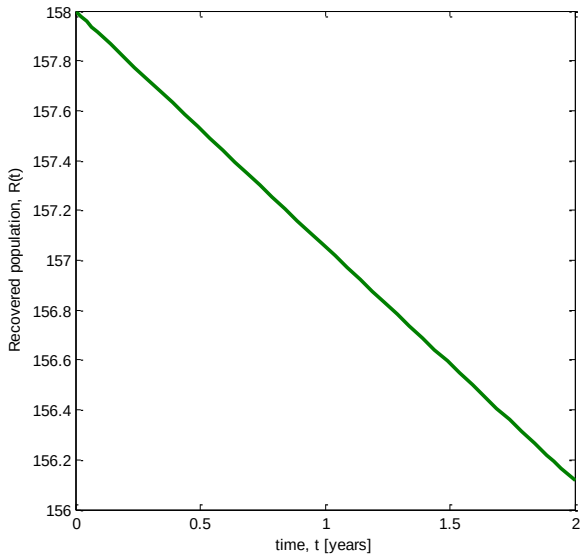

Figure 6. Population of Recovered Individuals, R (t) without control within two years, i.e. $t \in [0, 2.00]$ years

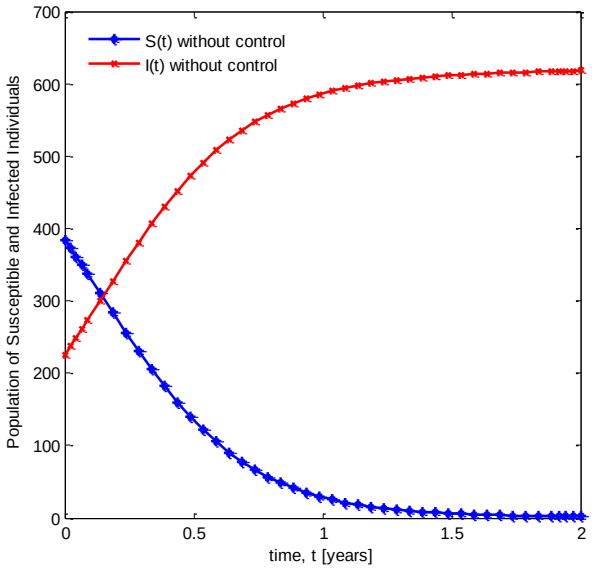

Figure 7. Population of both Susceptible and Infected Individuals, I (t) without control in two years, i.e. $t \in [0, 2.00]$ years.

Figure 7 shows that as the susceptible and infectious rate of onychomycosis increases without treatment, the population increases. This depict that the infection rate has direct effect on the individuals as the increase in infection is drastic.

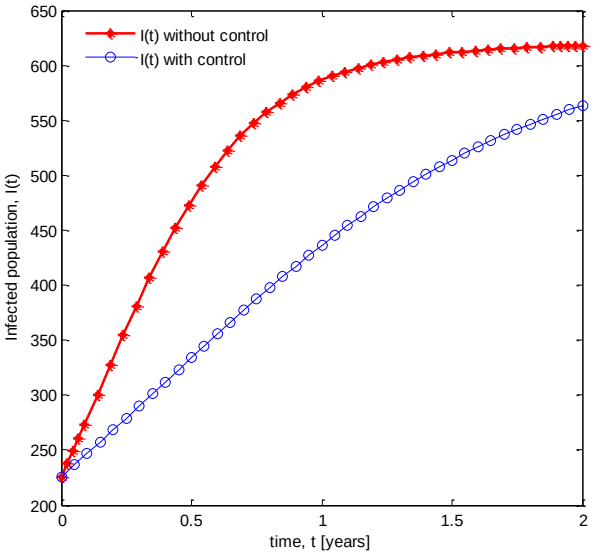

Figure 8. Population of Infected Individuals, I (t) without control and with control in two years, i.e. $t \in [0, 2.00]$ years

From figure 8, it shows that as the infectious rate of onychomycosis increases without treatment, the population increases. This depict that the infection rate has direct effect on the individuals as the increase in infection is drastic.

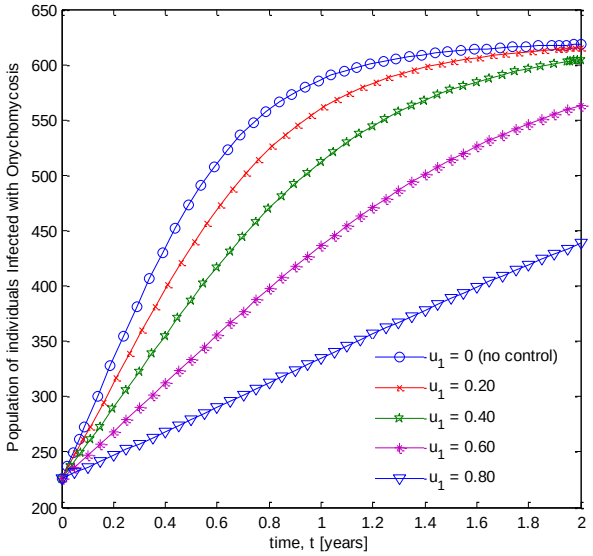

Figure 9. Impart of prevention control, u_1 on the population of infected individuals, I (t) in two years, i.e. $t \in [0, 2.00]$ years

Figure 9 shows that the rate of infection declined drastically in the population as Cymbopogon citratus extracts and essential oil was administered using a sterilized nail polish brush over a period of two years, the nails were checked periodically for less thickness, color improvement and less pain; the intervals of C. citratus application was two times a week consistently for two months, four months, six months, eight months to two years depending on the severity of onychomycosis infection. More so if there is no more interaction like using shared nail equipments, shoes or other environmental factors as contact with pets, walking

barefoot on the contaminated environment, there will be reduction in the exposed and susceptible individuals and this will in turn control infection rate.

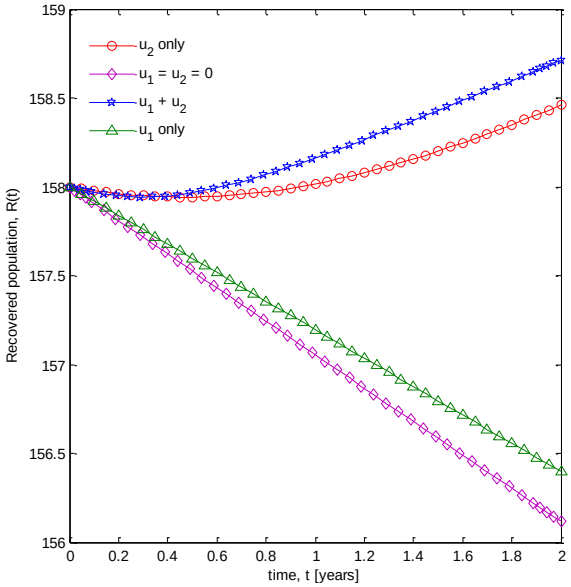

Figure 10. Impart of prevention and treatment controls, u_1 on the population of infected individuals, I (t) in two years, i.e. $t \in [0, 2.00]$ years

Figure 10 shows the transmission dynamics of onychomycosis with the presence of control measures (prevention and treatment controls). This shows that as the recovery rate increases, the population infected with onychomycosis decreases. This means that as people recover from onychomycosis due to consistent *C. citratus* extracts treatment within two, four, six and eight months, then a total recovery at one to two years, there will be no need to still be on treatment, as the nails shows clear improvement in its texture, size and color and appearance.

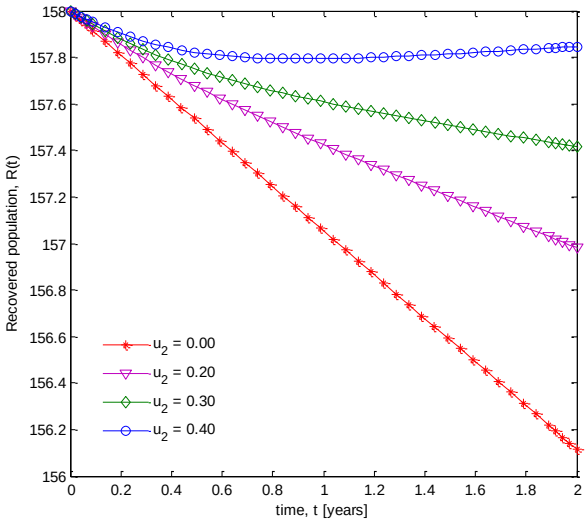

Figure 11. Impart of treatment controls, u_2 on the population of recovered individuals, R (t) in two years, i.e. $t \in [0, 2.00]$ years

From the figure 11, it shows that as the treatment rate increases within the first two, three and four months, the population of the susceptible class decreases. The susceptible

population decreases and attained an equilibrium position, due to enlightenment campaign, encouraging people to take adequate measures and care for their nails, wear less tightly fitted shoes and other environmental factors against the disease and to avoid exposing toe and fingernails to water, moist environment and shared nail care objects.

3.2. Discussion

This study on onychomycosis is a novel model, as there are no existing mathematical models in the past against the disease. The findings indicate that there is a positive correlation between the infection rate and the size of the susceptible infectious class. Furthermore, as the treatment with *C. citratus* increases, the size of the susceptible population decreases. This finding corroborated the claims made by Mutua et al. (2015) and Nthiiri (2016) whose study was on the model of Typhoid and it vaccination effect. This study's findings also revealed a positive correlation between the susceptible rate and population growth. As the rate of recovery increases, there is a corresponding decrease in the population of individuals with onychomycosis.

This finding corroborated with Kalajdzievska & Li (2011) who developed a mathematical model of the effects of carriers on the transmission dynamics of infectious diseases. They investigated that infections could be transmitted through carriers, infected individuals who are contagious but do not show any disease systems. It was assumed that the disease carriage state is infectious while those in the latent period are not. Their model incorporated demography and disease-induced death and it allows carriers to become symptomatic over time. They carried out local stability on the disease-free equilibrium. Their result 15 showed that a greater probability to develop carriage will increase the basic reproduction number which makes the infection persist. Testing and Diagnosis of carriers were seen as an effective control measure in a country where infectious diseases persist. Through mathematical analysis, Mutua et al. (2015) developed a mathematical model for malaria and typhoid fever coinfection dynamics. They first developed a model for only typhoid fever, their model 18 subdivides the human population of interest into four compartments susceptible humans(S), infected human (I), carrier humans (C), and recovered human (R). They later considered the incorporation of an additional compartment B, which represents bacteria in the environment. They identified distinct features of typhoid and malaria infection dynamics as well as the associated relationships.

The implementation of *C. citratus* (lemongrass) treatment for onychomycosis faces several practical challenges. The two-year treatment duration with twice-weekly applications may be difficult for patients to maintain consistently. Supply chain issues could affect the reliable availability of quality *C. citratus* extracts, while the costs of production and distribution might impact affordability for patients. Healthcare providers would require training in treatment protocols, and patients would need education about proper application and the importance of completing the full course. These barriers, while significant, could be addressed through careful planning, establishment of proper infrastructure, and development of comprehensive training programs.

4. Conclusion

In this study, a mathematical model was developed to understand the spread and control of onychomycosis by incorporating the treatment effects of *Cymbopogon citratus* (C. citratus) extracts. The equilibrium states of the model were derived and analyzed, and semi-analytical solutions were obtained using the Differential Transformation method. These solutions were visualized through simulations conducted with MATLAB and Maple. The numerical

simulations revealed that reducing contact rates with infectious individuals significantly lowers the transmission rate of the disease within the population. Additionally, consistent treatment of infected individuals with *C. citratus* extracts led to an increase in the number of recovered individuals, ultimately contributing to the eradication of onychomycosis. Furthermore, adherence to proper personal care of toe and fingernails, while considering environmental factors, was found to decrease the population of susceptible and exposed individuals.

Policymakers and health practitioners are encouraged to raise awareness about personal care of toe and fingernails, as the model indicates that transmission largely depends on contact with environmental microbes. Infected and treated individuals should be retested after recovery to prevent relapse. Regular cleaning of nails, at two-week intervals, and the application of *C. citratus* extracts are recommended as preventive measures, treatment options, and for maintaining nail health. One limitation of this study was the lack of comprehensive records of onychomycosis cases, so it is recommended that health workers maintain proper records to facilitate future research.

This study contributes to knowledge by formulating and validating a mathematical model for the transmission dynamics and control of onychomycosis, integrating *C. citratus* extracts as a treatment option. The findings demonstrate that the disease-free equilibrium is stable and that infected individuals can recover when treatment rates and efficacy are sufficiently high.

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